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Escape over fluctuating potential barrier with complicated dichotomous noise

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Abstract

In this paper, we report a kind of complicated dichotomous noise and investigate the escape over the fluctuating potential barrier for the one-dimensional process driven by this 'complicated dichotomous noise'. The study shows that the mean first passage time for a particle over the fluctuating potential barrier exhibits the resonant activation as the function of the transition rate of the complicated dichotomous noise. The effect of the parameters of this complicated dichotomous noise on the resonant activation is studied. In addition, a kind of more complicated dichotomous noise is introduced in the appendices.

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1. Introduction

Recently the conventional problems of the escape over the fluctuating potential barrier have attracted a great deal of attention [1–20]. It was shown that the mean first passage time (MFPT) of a particle driven by additive noises over a fluctuating potential barrier exhibits a minimum as a function of the flipping rate of the fluctuating potential barrier [1–20] (or the transition rate of the dichotomous noise). This phenomenon is called 'resonant activation', and first identified by Doering and Gadoua [1] and further studied by a number of other authors [2–20]. Now, except for the theoretical reports about the resonance activation phenomenon [1–20], this phenomenon was found in experiments [21–23].

Earlier studies of activation of MFPT over fluctuating potentials were restricted to limiting cases, i.e., slow [24] or fast [25, 26] barrier fluctuations, or small fluctuations [19]. Owing to using approximate treatments in [24–26], the resonant activation cannot be observed. Recently in [1–20], the authors reported results concerning the escape time (i.e. MFPT) over a fluctuating potential in the absence of approximate treatments as in [24–26]. They revealed the resonant activation (RA) of MFPT over the fluctuating potential barrier.

In this paper, we will report a kind of complicated dichotomous noise and investigate the escape over the fluctuating potential barrier for the one-dimensional process driven by this 'complicated dichotomous noise'. The setup of the problem is as follows: we first introduce a new kind of 'complicated dichotomous noise', and consider one-dimensional process driven by the complicated dichotomous noise and derive the master equation for the process. Then, using the master equation, we will study the escape over the fluctuating potential barrier. In addition, a new kind of more complicated dichotomous noise will be introduced in the appendices.

2. Complicated dichotomous noise

When dealing with the dichotomous noise, we usually believe that it takes two different constant values. In this section, we report a kind of dichotomous noise whose one or two value is Gaussian white noise. We call this kind of dichotomous noise 'complicated dichotomous noise'.

Case I. A kind of dichotomous noise whose one value is Gaussian white noise.

In this case, we assume that the dichotomous noise $\xi(t)$ takes values a and $\eta(t)$, where a is a constant , and $\eta(t)$ a Gaussian white noise with zero mean and correlation function $\langle\eta(t)\eta(t')\rangle = 2D_1\delta(t-t')$. The transition rates for $\xi(t)$ from a to $\eta(t)$ or vice versa are respectively, μ and μ'. The master equation for the probability density of $\xi(t)$ is

$$\begin{aligned} \partial_t P(\eta,t) &= -\mu' P(\eta,t) + \mu P(a,t), \\ \partial_t P(a,t) &= -\mu P(a,t) + \mu' P(\eta,t). \end{aligned} \tag{1}$$

Case II. A kind of dichotomous noise whose both values are Gaussian white noises.

Now, we assume that the dichotomous noise $\xi(t)$ takes values $\eta_1(t)$ and $\eta_2(t)$, in which $\eta_1(t)$ and $\eta_2(t)$ are Gaussian white noises with zero means and correlation functions $\langle\eta_1(t)\eta_1(t')\rangle = 2D_1\delta(t-t')$, $\langle\eta_2(t)\eta_2(t')\rangle = 2D_2\delta(t-t')$ and $\langle\eta_1(t)\eta_2(t')\rangle = 0$. The transition rates for $\xi(t)$ from $\eta_1(t)$ to $\eta_2(t)$ and vice versa are respectively μ and μ'. The master equation for the probability density of $\xi(t)$ is

$$\begin{aligned} \partial_t P(\eta_1,t) &= -\mu P(\eta_1,t) + \mu' P(\eta_2,t), \\ \partial_t P(\eta_2,t) &= -\mu' P(\eta_2,t) + \mu P(\eta_1,t). \end{aligned} \tag{2}$$

Above, we introduce the complicated dichotomous noise, below we will consider one-dimensional process driven by complicated dichotomous noise, and derive the master equation for the process. Then, using the master equation, we will study the escape over the fluctuating potential barrier.

3. One-dimensional process driven by the complicated dichotomous noise and the master equation

In this section, we consider a one-dimensional process subject to the complicated dichotomous noise. The Langevin equation of this process is

$$\dot{x} = f(x) + g(x)\xi(t) + \zeta(t), \tag{3}$$

where $f(x)$ and $g(x)$ are deterministic functions of x, $\xi(t)$ is the complicated dichotomous noise, and $\zeta(t)$ a Gaussian white noise with zero mean and correlation function $\langle\zeta(t)\zeta(t')\rangle = 2D\delta(t-t')$. There is no correlation between $\xi(t)$ and $\zeta(t)$. $\xi(t)$ is the same as given in section 2.

3.1. A kind of the dichotomous whose one value is Gaussian white noise

When the dichotomous noise $\xi(t)$ takes the constant value a, equation (3) becomes

$$\dot{x} = f(x) + ag(x) + \zeta(t). \tag{4}$$

The Fokker–Planck equation (FPE) for equation (4) is

$$\partial_t P'(x,t) = -\partial_x[f(x) + ag(x)]P'(x,t) + D\partial_x^2 P'(x,t). \tag{5}$$

When the dichotomous noise $\xi(t)$ takes the Gaussian white noise $\eta(t)$, equation (3) becomes

$$\dot{x} = f(x) + g(x)\eta(t) + \zeta(t). \tag{6}$$

The FPE of equation (6) is

$$\partial_t P''(x,t) = -\partial_x f(x)P''(x,t) + D_1\partial_x g(x)\partial_x g(x)P''(x,t) + D\partial_x^2 P''(x,t). \tag{7}$$

Since now there is a joint process about x, $\xi(t)$ and $\zeta(t)$ for equation (3), the master equation of equation (3) is

$$\begin{aligned}\partial_t P_1 &= -\mu P_1 + \mu' P_2 - \partial_x[f(x) + ag(x)]P_1 + D\partial_x^2 P_1,\\ \partial_t P_2 &= -\mu' P_2 + \mu P_1 - \partial_x f(x)P_2 + D_1\partial_x g(x)\partial_x g(x)P_2 + D\partial_x^2 P_2,\end{aligned} \tag{8}$$

where $P_1 = P(x,t,a)$ and $P_2 = P(x,t,\eta)$. $P(x,t,a)$ represents that the particle is at x, and the noise $\xi(t)$ in $\xi(t) = a$ configuration, and $P(x,t,\eta)$ does that the particle is at x and the noise $\xi(t)$ in $\xi(t) = \eta(t)$ configuration. The probability density for equation (3) is $P(x,t) = P_1 + P_2$.

3.2. A kind of the dichotomous whose both values are Gaussian white noises

We consider a process whose Langevin equation is also equation (3). If the dichotomous noise $\xi(t)$ only takes $\eta_1(t)$, the FPE corresponding to equation (3) is $\partial_t P_1 = -\partial_x f(x)P_1 + D_1\partial_x g(x)\partial_x g(x)P_1 + D\partial_x^2 P_1$. If $\xi(t)$ only takes $\eta_2(t)$, the FPE is $\partial_t P_2 = -\partial_x f(x)P_2 + D_2\partial_x g(x)\partial_x g(x)P_2 + D\partial_x^2 P_2$. So, the master equation for this process is

$$\begin{aligned}\partial_t P_1 &= -\mu P_1 + \mu' P_2 - \partial_x f(x)P_1 + D_1\partial_x g(x)\partial_x g(x)P_1 + D\partial_x^2 P_1,\\ \partial_t P_2 &= -\mu' P_2 + \mu P_1 - \partial_x f(x)P_2 + D_2\partial_x g(x)\partial_x g(x)P_2 + D\partial_x^2 P_2.\end{aligned} \tag{9}$$

4. Escape over fluctuating potential barrier

In this section, we use the master equation (8) [or (9)] to deal with the problem of the escape over the fluctuating potential barrier [1–26]. The Langevin equation of this problem is

$$\dot{x} = -\partial_x U(x) + \xi(t) + \zeta(t), \tag{10}$$

in which $U(x)$ is the potential, which is a symmetric piecewise linear ratchet (see figure 1). $\xi(t)$ and $\zeta(t)$ are the same as above. The fluctuating potential barrier $U(x,t)$ satisfies

$$\partial_x U(x,t) = \partial_x U(x) - \xi(t).$$

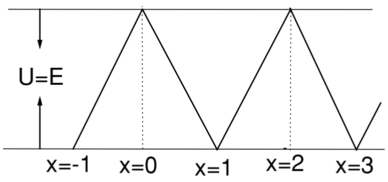

Figure 1. The symmetric piecewise linear ratchet potential barrier. For the numerical simulation in this paper, we take $L = 2$.

4.1. Case I: one value of $\xi(t)$ is Gaussian white noise

According to equation (8), the master equation of equation (10) is ($\mu = \mu'$)

$$\begin{aligned} \partial_t P_1 &= -\mu P_1 + \mu P_2 + (E - a)\partial_x P_1 + D\partial_x^2 P_1, \\ \partial_t P_2 &= -\mu P_2 + \mu P_1 + E\partial_x P_2 + (D + D_1)\partial_x^2 P_2. \end{aligned} \tag{11}$$

We assume that the particle starts at $x = -1$. So, the initial condition is $P_i(x, 0) = \frac{1}{2}\delta(x + 1)$. The reflecting boundary condition is $\partial_x P_i(x, t)|_{x=-1} = 0$, and the absorbing boundary condition $P_i(x, t)|_{x=0} = 0$. The backward master equation for equation (11) is [27]

$$\begin{aligned} \partial_t G_1 &= -\mu G_1 + \mu G_2 - (E - a)\partial_x G_1 + D\partial_x^2 G_1, \\ \partial_t G_2 &= -\mu G_2 + \mu G_1 - E\partial_x G_2 + (D + D_1)\partial_x^2 G_2. \end{aligned} \tag{12}$$

The mean first passage time (MTPT) is defined as [27]

$$T_i(x) = -\int_0^\infty t\partial_t G_i(x, t)\,\mathrm{d}t = \int_0^\infty G_i(x, t)\,\mathrm{d}t, \tag{13}$$

where $i = 1$ and 2.

From equations (12) and (13), we can obtain the MFPT equations for equation (10) as follows:

$$\begin{aligned} &-\mu T_1 + \mu T_2 - (E - a)\partial_x T_1 + D\partial_x^2 T_1 + 1/2 = 0, \\ &-\mu T_2 + \mu T_1 - E\partial_x T_2 + (D + D_1)\partial_x^2 T_2 + 1/2 = 0. \end{aligned} \tag{14}$$

The reflecting boundary condition and the absorbing boundary condition are respectively $\partial_x T_i(-1) = 0$, $T_i(0) = 0$. The MFPT for a particle over the fluctuating potential barrier that starts at $x = -1$ is $T = \sum_{i=1}^{2} T_i(-1)$.

Generally, we cannot get the exact expression of the MFPT. But, in the case when $2E - a = 0$, it is simple enough to summarize analytically. In this case, the MFPT for a particle over a fluctuating potential barrier is explicitly

$$\begin{aligned} T = {}& \big(1 - k_1^{(2)}\big)A_1^{(1)} \exp(-\lambda_1)/\lambda_1 + \big(1 - k_2^{(2)}\big)A_2^{(1)} \exp(-\lambda_2)/\lambda_2 + 2B_3^{(1)} \\ & + M_2 + P_2 A_3^{(1)} + q_2 - 2A_3^{(1)} - N_2 - \mu/[(2D + D_1)\mu + E^2], \end{aligned} \tag{15}$$

where

$$\lambda_{1,2} = \big[-ED_1 \pm \sqrt{E^2(2D + D_1)^2 + 4\mu(D^2 + DD_1)(2D + D_1)}\big]/[2D(D + D_1)],$$

$$A_1^{(1)} = \frac{M_2 + q_2 - \mu P_2/[(2D + D_1)\mu + E^2] - N_2\big[\big(1 - k_2^{(2)}\big)/\lambda_2 + P_2 \exp(-\lambda_2)\big/\big(1 - k_2^{(2)}\big]}{\big(1 - k_1^{(2)}\big)/\lambda_1 + P_2 \exp(-\lambda_1) - \big(1 - k_1^{(2)}\big)\exp(\lambda_2 - \lambda_1)/\lambda_2 - P_2\big(1 - k_1^{(2)}\big)\exp(-\lambda_1)/\big(1 - k_2^{(2)}\big)},$$

$$A_2^{(1)} = -A_1^{(1)}\big(1 - k_1^{(2)} \exp(-\lambda_1 + \lambda_2)\big/\big(1 - k_2^{(2)}\big) + N_2 \exp(\lambda_2)\big/\big(1 - k_2^{(2)},$$

$$A_3^{(1)} = -A_1^{(1)} \exp(-\lambda_1) - A_2^{(1)} \exp(-\lambda_2) - \mu/[(2D + D_1)\mu + E^2],$$

and

$$B_3^{(1)} = -A_1^{(1)}/\lambda_1 - A_2^{(1)}/\lambda_2,$$

with

$$k_j^1 = 1, \quad K_j^{(2)} = 1 + E\lambda_j/\mu - (D + D_1)\lambda_j^2/\mu (j = 1, 2), \quad N_1 = 0,$$
$$N_2 = -E/[(2D + D_1)\mu + E^2], \quad M_1 = 0,$$
$$M_2 = (D + D_1)/[(2D + D_1)\mu + E^2], \quad P_2 = E/\mu,$$

and $q_2 = -1/(2\mu)$.

When $2E - a \neq 0$, we cannot analytically obtain the exact expression of the MFPT. Now, taking $\partial_x T_i = s_{(i)}$ (i = 1, 2), equation (14) becomes

$$\partial_x \begin{pmatrix} s_1 \\ T_1 \\ s_2 \\ T_2 \end{pmatrix} = \begin{pmatrix} \frac{E}{D+D_1} & \frac{\mu}{D+D_1} & 0 & \frac{\mu'}{D+D_1} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{-\mu}{D} & \frac{E-a}{D} & \frac{\mu}{D} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s_1 \\ T_1 \\ s_2 \\ T_2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2(D+D_1)} \\ 0 \\ -\frac{1}{2D} \\ 0 \end{pmatrix}. \qquad (16)$$

By numerical simulation and analysis we can find that when $2E - a \neq 0$ the matrix of the homogeneous part about T_i and s_i $(i = 1, 2)$ in equation (16) has three nonzero real independent eigenvalues and a zero eigenvalue. So, the general solution of equation (16) is

$$s_i = \sum_{j=1}^{3} A_j^{(i)} \exp(\lambda_j x) + A_4^{(i)} + A_5^{(i)} x, \qquad T_i = \sum_{j=1}^{3} B_j^{(i)} \exp(\lambda_j x) + B_4^{(i)} + B_5^{(i)} x, \qquad (17)$$

where $i = 1{,}2$, λ_j $(j = 1, 2, 3)$ is the above-mentioned nonzero eigenvalues. Substituting equation (17) into equation (16) and using the comparing-coefficient method, we obtain $B_j^{(i)} = A_j^{(i)}/\lambda_j$, $A_5^{(i)} = 0$, $B_5^{(i)} = A_4^{(i)}$, $A_4^{(1)} = A_4^{(2)} = \frac{1}{2E-a}$, $B_4^{(i)} = B_4^{(1)} + F_i$, and $A_j^{(i)} = K_j^{(i)} A_j^{(1)}$, with $F_1 = 0$, $F_2 = \frac{a}{2\mu(2E-a)}$, $K_j^{(1)} = 1$, and $K_j^{(2)} = -(D + D_1)\lambda_j^2/\mu + 1 + E\lambda_j/\mu$. So, equation (17) becomes

$$s_i = \sum_{j=1}^{3} K_j^{(i)} A_j^{(1)} \exp(\lambda_j x) + \frac{1}{2E - a},$$
$$T_i = \sum_{j=1}^{3} K_j^{(i)} A_j^{(1)} \frac{1}{\lambda_j} \exp(\lambda_j x) + B_4^{(1)} + F_i + \frac{1}{2E - a} x. \qquad (18)$$

Then, substituting equation (18) into the boundary conditions $T_i(0) = 0$ and $s_i(-1) = 0$, we can obtain a linear algebraic system for $A_j^{(1)} (j = 1, 2, 3)$ and $B_4^{(1)}$. From the linear algebraic equations of this algebraic system we can derive $A_j^{(1)}$ and $B_4^{(1)}$. The MFPT for a particle over the fluctuating barrier is

$$T = \sum_{i=1}^{2} T_i(-1) = \sum_{i=1}^{2} \sum_{j=1}^{3} \frac{K_j^{(i)} A_j^{(1)}}{\lambda_j} \exp(-\lambda_j) + 2B_4^{(1)} + F_2 - \frac{2}{2E - a}. \qquad (19)$$

In figures 2 and 3, the ln of the MFPT versus the ln of the transition rate μ of the complicated dichotomous noise is plotted when $2E - a = 0$ and $2E - a \neq 0$, respectively. Figures 2(*a*) and 3(*a*) correspond to the ln of T versus the ln of μ for different values of a, figures 2(*b*) and 3(*b*) to that for different values of D_1. The figures show that the MFPT of the particle over the fluctuating potential barrier exhibit a minimum as a function of the transition rate of the complicated dichotomous noise. This phenomenon has been reported in [1–23], which has been named as 'resonant activation'. A reason for the resonant activation (RA) happening here is given below. The resonance in figures 2(*a*), (*b*), 3(*a*) and (*b*) occurs

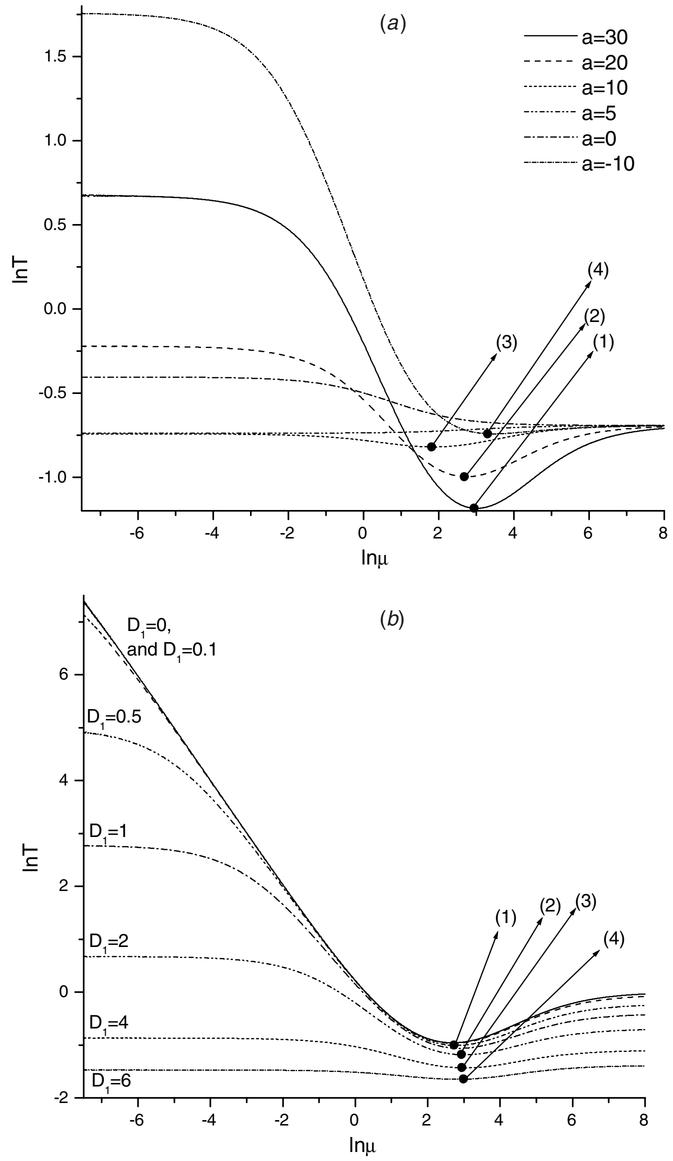

Figure 2. The ln of the MFPT versus the ln of the transition rate μ of the complicated dichotomous noise when $2E - a = 0$ for model (10) in case I. Part (*a*) corresponds to the ln of the MFPT versus the ln of μ for different values of a (a = 30, 20, 10, 5, 0 and -10) with $D = 1$ and $D_1 = 2$; (*b*) to that for different values of D_1 (D_1 = 0, 0.1, 0.5, 1, 2, 4 and 6) with $D = 1$ and $a = 30$. The marked points (1)–(4) in figure 2(*a*) and points (1)–(4) in 2(*b*) are the points where the transition time equals the MFPT over the fluctuating barrier with the effective potential barrier in $U = \min(E - a, \frac{E}{1+D_1/D})$ configuration.

when the crossing takes place with the fluctuation effective potential barrier[1] most likely being in $U = \min\left(E - a, \frac{E}{1+D_1/D}\right)$ configuration (i.e., the 'down' configuration). Now the MFPT

[1] For equation (11), we define the effective potential $U_1^{\mathrm{eff}} = \frac{E-a}{D} D = E - a$ of the first equation; then, for the second equation the effective potential is $U_2^{\mathrm{eff}} = \frac{E}{D+D_1} D = \frac{E}{1+D_1/D}$. So, for equation (11), now there are two configurations, $U_1^{\mathrm{eff}} = E - a$ and $U_2^{\mathrm{eff}} = \frac{E}{1+D_1/D}$.

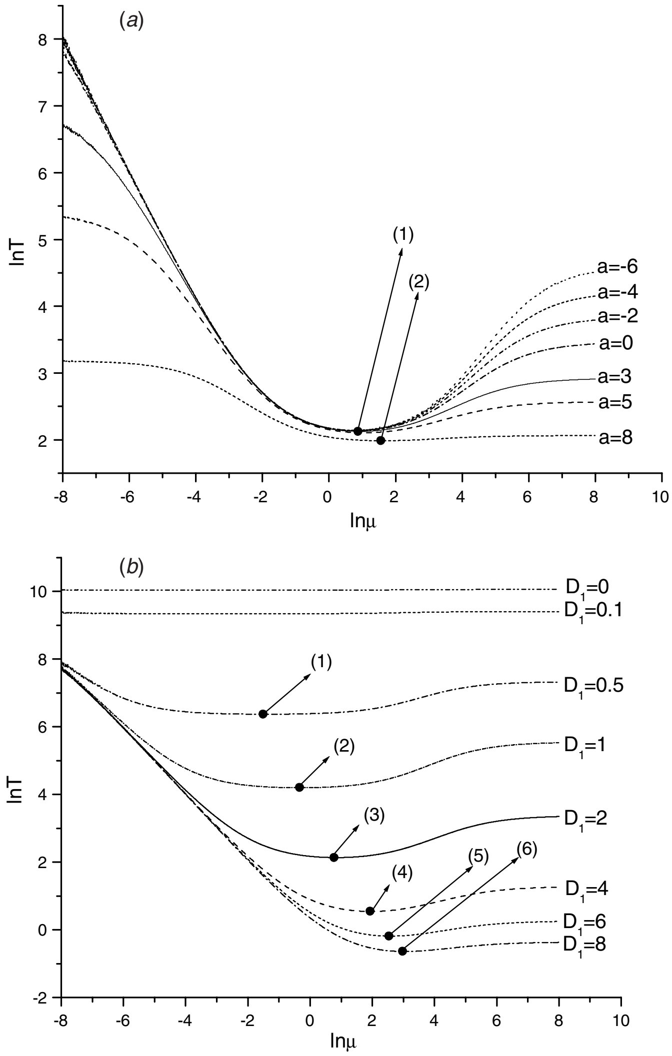

Figure 3. The ln of the MFPT versus the ln of the transition rate μ of the complicated dichotomous noise when $2E - a \neq 0$ for model (10) in case I. Part (a) corresponds to the ln of the MFPT versus the ln of μ for different values of a ($a = -6, -4, -2, 0, 3, 5$ and 8) with $D = 1$, $D_1 = 2$ and $E = 15$; (b) to that for different values of D_1 ($D_1 = 0, 0.1, 0.5, 1, 2, 4, 6$ and 8) with $D = 1$, $E = 15$ and $a = 0.5$. The marked points (1) and (2) in figure 3(a) and points (1)–(6) in figure 3(b) are the points where the transition time equals the MFPT over the fluctuating barrier with the potential barrier in $U = \min(E - a, \frac{E}{1+D_1/D})$ configuration.

has a minimum for the fluctuating potential barrier transition rate of the order of the inverse of the time required to cross the potential with the fluctuation effective potential barrier in $U = \min\left(E - a, \frac{E}{1+D_1/D}\right)$ configuration. In figures 2(a), (b), 3(a) and (b), we plot some points where the transition time equals the MFPT over the fluctuating potential barrier with the fluctuation effective potential barrier in $U = \min\left(E - a, \frac{E}{1+D_1/D}\right)$ configuration. It is clear that this accords with the above reason for the RA happening in figures 2(a), (b), 3(a) and (b). When the resonant activation emerges, the fluctuation effective potential is rarely probably in

$U = \max\left(E - a, \frac{E}{1+D_1/D}\right)$ configuration (now the fluctuation effective potential is most likely in $U = \min\left(E - a, \frac{E}{1+D_1/D}\right)$ configuration). In addition, we find that, when $2E - a = 0$, the absolute value of a can enhance the RA with the increase of the absolute values of a, but D_1 can weaken it with the increase of the values of D_1, which can be observed from figures 2(a) and (b). When $2E - a \neq 0$, the negative value of a can enhance the RA, while the positive value of a can weaken it, with the increase of the absolute values of a, which can be seen from figure 3(a); in figure 3(b), we can observe that the good RA effect is for $D_1 = 2$, which corresponds approximately to the behaviour of figure 3(a) for $a = 0$, in some sense this $D_1 = 2$ should be an optimal value to observe RA for the parameter values considered.

4.2. Case II: two values of $\xi(t)$ are Gaussian white noises

Now, according to equation (9), the master equation of equation (10) is ($\mu = \mu'$)

$$\begin{aligned} \partial_t P_1 &= -\mu P_1 + \mu P_2 + E\partial_x P_1 + (D + D_1)\partial_x^2 P_1, \\ \partial_t P_2 &= -\mu P_2 + \mu P_1 + E\partial_x P_2 + (D + D_2)\partial_x^2 P_2. \end{aligned} \tag{20}$$

Similarly, we assume that the particle starts at $x = -1$. The boundary conditions for equation (20) are the same as for equation (11).

The equations for the MFPT over the fluctuating barrier is

$$\begin{aligned} -\mu T_1 + \mu T_2 - E\partial_x T_1 + (D + D_1)\partial_x^2 T_1 + 1/2 = 0, \\ -\mu T_2 + \mu T_1 - E\partial_x T_2 + (D + D_2)\partial_x^2 T_2 + 1/2 = 0. \end{aligned} \tag{21}$$

The boundary conditions for equation (21) are $\partial_x T_i(-1) = 0$ and $T_i(0) = 0$. The MFPT for a particle over the fluctuating potential barrier is $T = \sum_{i=1}^{2} T_i(-1)$.

Taking $\partial_x T_i = s_i$, equation (21) can be written as

$$\partial_x \begin{pmatrix} s_1 \\ T_1 \\ s_2 \\ T_2 \end{pmatrix} = \begin{pmatrix} \frac{E}{D+D_1} & \frac{\mu}{D+D_1} & 0 & \frac{\mu}{D+D_1} \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\mu}{D+D_2} & \frac{E}{D+D_2} & \frac{\mu}{D+D_2} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s_1 \\ T_1 \\ s_2 \\ T_2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2(D+D_1)} \\ 0 \\ -\frac{1}{2(D+D_2)} \\ 0 \end{pmatrix}. \tag{22}$$

Numerical simulation and analysis show that the eigenvalues of the matrix of the homogeneous part about T_i and s_i (i = 1, 2) in equation (22) are real and independent, and there is a zero eigenvalue. Using the method for deriving equation (18), we can get the general solution of equation (22). It is

$$\begin{aligned} s_i &= \sum_{j=1}^{3} K_j^{(i)} A_j^{(1)} \exp(\lambda_j x) + \frac{1}{E}, \\ T_i &= \sum_{j=1}^{3} \frac{1}{\lambda_j} K_j^{(i)} A_j^{(1)} \exp(\lambda_j x) + \frac{x}{E} + B_4^{(1)}, \end{aligned} \tag{23}$$

where $i = 1, 2$, $K_j^{(1)} = 1$ and $K_j^{(2)} = 1 + E\lambda_j/\mu - (D + D_1)\lambda_j^2/\mu$. $\lambda_j (j = 1, 2, 3)$ are three real independent nonzero eigenvalues of the matrix of the homogeneous part in equation (22).

Substituting equation (23) into the boundary conditions $T_i(0) = 0$ and $s_i(-1) = 0$ ($i = 1, 2$), one can get four linear algebraic equations for $A_j^{(1)}$ ($j = 1, 2, 3$) and $B_4^{(1)}$. From these linear algebraic equations, one can obtain $A_j^{(1)}$ and $B_4^{(1)}$. The MFPT for a particle over the fluctuating barrier is

$$T = \sum_{i=1}^{2} T_i(-1) = \sum_{i=1}^{2}\sum_{j=1}^{3} \frac{K_j^{(i)} A_j^{(1)}}{\lambda_j} \exp(-\lambda_j) + 2B_4^{(1)} - \frac{2}{E}. \tag{24}$$

In figures 4 and 5, we plot the ln of the MFPT as the function of the transition rate μ of the complicated dichotomous noise for different values of D_1 (figure 4(a) corresponds to $D_1 = 0$, 0.5, 1, 2 and 2.7 with $D = 1$, $D_2 = 3$ and $E = 15$, figure 4(b) to $D_1 = 3.3$, 5, 7, 9 and 14 with $D = 1$, $D_2 = 3$ and $E = 15$), and for different values of D_2 (figure 5(a) corresponds to $D_2 = 0$, 0.2, 0.6, 0.9 and 0.95 with $D = 1$, $D_1 = 1$ and $E = 15$, figure 5(b) to $D_2 = 1.05$, 3, 7, 9 and 14 with $D = 1$, $D_1 = 1$ and $E = 15$), respectively. From the figures, we can find that there is the RA for the MFPT over the fluctuating potential barrier as the function of the transition rate μ. The resonance for the RA occurs when the crossing takes place with the fluctuation effective potential barrier[2] most likely in $U = \min\left(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D}\right)$ configuration. Now the MFPT has a local minimum for the fluctuation potential barrier transition rate of the order of the inverse of the time required to cross the fluctuation barrier with the fluctuation effective potential barrier in $U = \min\left(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D}\right)$ configuration. In figures 4(a), (b), 5(a) and (b), we plot some points where the transition time equals the MFPT over the fluctuating barrier with the effective potential barrier in $U = \min\left(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D}\right)$ configuration. When the resonant activation emerges, the fluctuation effective potential is rarely probably in $U = \max\left(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D}\right)$ configuration (now the fluctuation effective potential is most likely in $U = \min\left(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D}\right)$ configuration). Moreover, the study shows that, for definite values of D, D_2 and E, when $D_1 = 0$ the RA is the most distinct (see figure 4(a)); for definite values of D, D_1 and E, when $D_2 = 0$ the RA is the most distinct (see figure 5(a)).

For this kind of complicated dichotomous noise, when $D_1 = D_2 = D_0$, the complicated dichotomous noise $\xi(t)$ will become a Gaussian white noise $\xi_0(t)$ with zero mean and correlation function $\langle \xi_0(t)\xi_0(t')\rangle = 2D_0\delta(t - t')$. So, when $D_1 = D_2$, the MFPT will become $T = (\exp(r_1)/r_1 - 1/r_1 - 1)/r_1$ with $r_1 = E/(D + D_0)$ (which is not related to the transition rate μ). It is clear that now no RA exists. In figures 4(a) and (b), when $D_1 = D_2 = 3$, $\ln T \doteq 1.01$, which is marked in these figures; in figures 5(a) and (b), when $D_1 = D_2 = 1$, $\ln T \doteq 4.17$ (which is also marked in these figures).

5. Conclusion and discussion

In conclusion, we report a kind of 'complicated dichotomous noise' and investigate the escape over the fluctuating potential barrier for the one-dimensional process driven by the 'complicated dichotomous noise'. Study shows that the mean first passage time for a particle over the fluctuating potential barrier exhibits the resonant activation as the function of the transition rate of the 'complicated dichotomous noise'. The effect of the parameters of the 'complicated dichotomous noise' on the resonant activation is studied. The motivation for considering activation over fluctuating potential barriers was to study the models of relaxation in complex many-body systems.

I have noted that Dubkov, Agudov and Spagnolo obtained the resonant activation effect with a theoretical approach, without any approximation in the noise intensity, the parameter

[2] As in footnote 1, for equation (20), we define the effective potential $U_1^{\text{eff}} = \frac{E}{D-D_1} D = \frac{E}{1+D_1/D}$ of the first equation; then, for the second equation the effective potential is $U_2^{\text{eff}} = \frac{E}{D+D_2} D = \frac{E}{1+D_2/D}$. Thus, equation (20) now has two configurations, $U_1^{\text{eff}} = \frac{E}{1+D_1/D}$ and $U_2^{\text{eff}} = \frac{E}{1+D_2/D}$.

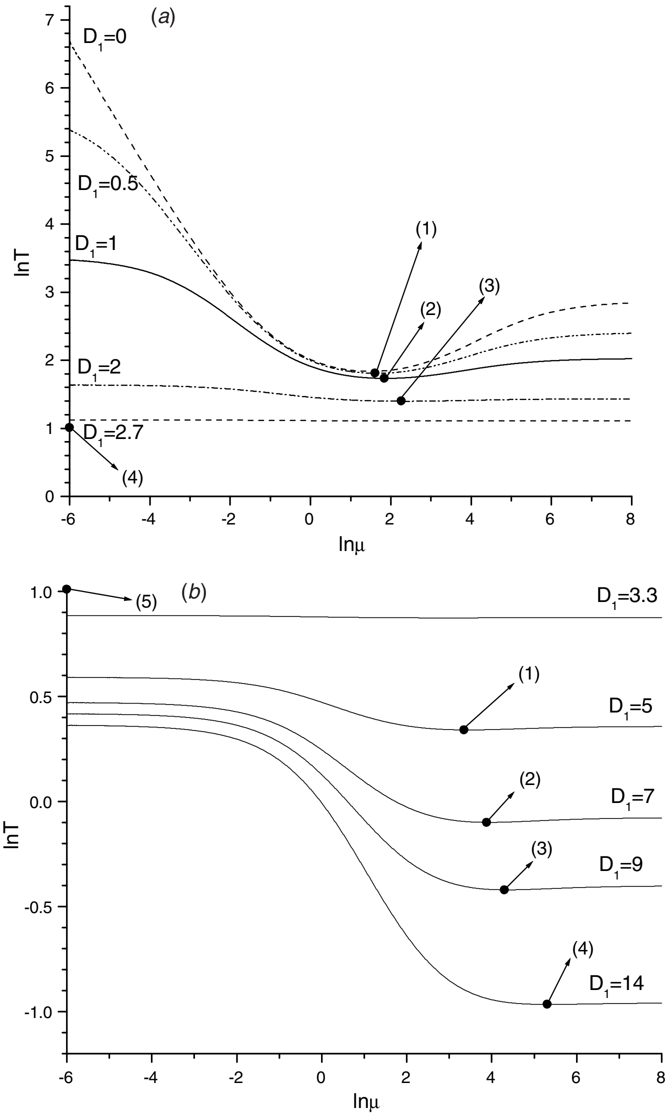

Figure 4. The ln of the MFPT versus the ln of the transition rate μ of the complicated dichotomous noise for different values of D_1 with $D = 1$, $D_2 = 3$ and $E = 15$ for model (10) in case II. Part (*a*) corresponds to the ln of the MFPT versus the ln of μ for $D_1 = 0, 0.5, 1, 2$ and 2.7; (*b*) to that for $D_1 = 3.3, 5, 7, 9$ and 14. The marked points (1), (2) and (3) in figure 4(*a*) and points (1)–(4) in figure 4(*b*) are the points where the transition time equals the MFPT over the fluctuating barrier with the potential barrier in $U = \min(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D})$ configuration. The marked point (4) in figure 4(*a*) and that (5) in figure 4(*b*) are the points where $D_1 = D_2 = 3$.

of the potential and the rate of the dichotomous noise [28, 29]. Specifically in [28] an enhancement of the escape time as a function of the mean flipping rate of the dichotomous noise was obtained together with the RA effect (see figure 5 in [28]). This peculiar behaviour was recently experimentally observed in a Josephson junction [23].

In [30], Horsthemke, Doering, Ray and Burschka investigated a reversible diffusion-limited-coagulation reaction $A + A \longleftrightarrow A$ with irreversible input $B \longrightarrow A$ in one spatial

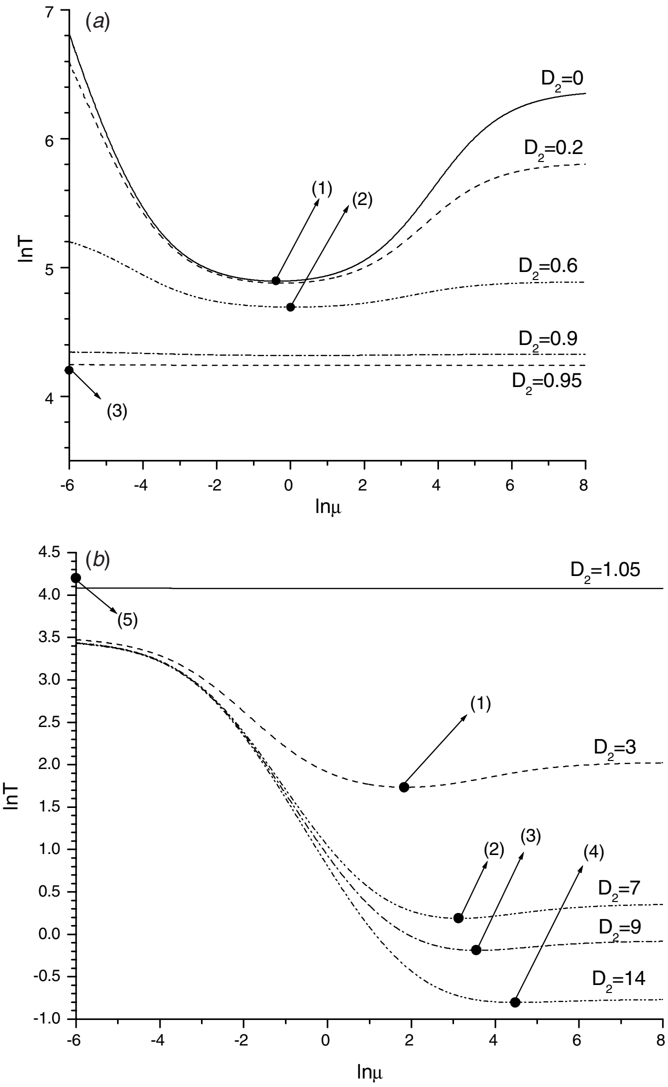

Figure 5. The ln of the MFPT versus the ln of the transition rate μ of the complicated dichotomous noise for different values of D_2 with $D = 1$, $D_1 = 1$ and $E = 15$ for model (10) in case II. Part (*a*) corresponds to the ln of the MFPT versus the ln of μ for $D_2 = 0, 0.2, 0.6, 0.9$ and 0.95; (*b*) to that for $D_2 = 1.05, 3, 7, 9$ and 14. The marked points (1) and (2) in figure 5(*a*) and points (1)–(4) in figure 5(*b*) are the points where the transition time equals the MFPT over the fluctuating barrier with the potential barrier in $U = \min(\frac{E}{1+D_1/D}, \frac{E}{1+D_2/D})$ configuration. The marked point (3) in figure 5(*a*) and that (5) in figure 5(*b*) are the points where $D_1 = D_2 = 1$.

dimension, and derived explicit results for the interparticle distribution function, i.e., the probability density for finding the nearest particle a distance x from a given particle, with external dichotomous noise in the birth rate, and in particular the limiting case that the birth rate fluctuates between zero and fixed positive values. But they have not considered the mean first passage time for the particle over the fluctuating potential barrier. If they had studied the mean first passage time for the particle over the fluctuating barrier, a phenomenon of resonant activation should have been found for the system they considered.

In this paper, we introduce a kind of 'complicated dichotomous noise'. But practically, there are a lot of more complicated dichotomous noises, such as the one given in the appendices (see the appendices). In the appendices, we introduce a kind of more complicated dichotomous noise. We call it 'more complicated dichotomous noise'.

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Appendix A. A kind of dichotomous noise whose one value is stochastic about *x* and *t*

We consider a dichotomous noise $\eta(x,t)$ which takes a stochastic value $\xi_1(x,t)$ and a variable value $a(x)$, where $\xi_1(x,t)$ is stochastic Gaussian variable with respect to x and t with zero mean and correlation function $\langle\xi_1(x,t)\xi_1(y,s)\rangle = 2w(x,y)\delta(t-s)$, and $a(x)$ a deterministic function of x. The transition rates for $\eta(x,t)$ from $\xi_1(x,t)$ to $a(x)$ and vice verse are μ and μ', respectively. The master equation for the noise $\eta(x,t)$ are

$$\begin{aligned}
&\partial_t P(t,\xi_1(x,t)) = -\mu P(t,\xi_1(x,t)) + \mu' P(t,a(x)),\\
&\partial_t P(t,a(x)) = -\mu' P(t,a(x)) + \mu P(t,\xi_1(x,t)),
\end{aligned} \tag{A.1}$$

In the following, we study a system with this dichotomous noise whose Langevin equation is

$$\dot{x} = f(x) + g(x)\eta(x,t), \tag{A.2}$$

in which $f(x)$ and $g(x)$ are determined function of x.

When the dichotomous noise only takes the stochastic value $\xi_1(x,t)$, equation (A.2) becomes

$$\dot{x} = f(x) + g(x)\xi_1(x,t). \tag{A.3}$$

To get the probability density equation of equation (A.3), we assume its probability density

$$P_2(x,t) = \langle\delta(x - x(t))\rangle, \tag{A.4}$$

where $x(t)$ is the solution of equation (A.2). Differentiating equation (A.4) with respect to t, we obtain

$$\begin{aligned}
\partial_t P_2(x,t) &= \langle\partial_t\delta(x-x(t))\rangle\\
&= -\left\langle\frac{\mathrm{d}x(t)}{\mathrm{d}t}\partial_x\delta(x-x(t))\right\rangle = -\langle[f(x(t)) + g(x(t))\xi_1(x,t)]\partial_x\delta(x-x(t))\rangle\\
&= -\partial_x f(x)P_2(x,t) - \partial_x\langle g(x)\xi_1(x,t)\delta(x-x(t))\rangle\\
&= -\partial_x f(x)P_2(x,t) - \partial_x\int\int_0^t \mathrm{d}x'\,\mathrm{d}t'\langle\xi_1(x,t)\xi_1(x',t')\rangle\left\langle\frac{\delta[g(x)\delta(x-x(t))]}{\delta\xi_1(x',t')}\right\rangle\\
&= -\partial_x f(x)P_2(x,t) - \partial_x\int\int_0^t \mathrm{d}x'\,\mathrm{d}t'2w(x,x')\delta(t-t')\\
&\quad\times\left[-\left\langle\frac{\delta x(t)}{\delta\xi_1(x',t')}g(x)\partial_x\delta(x-x(t))\right\rangle\right]\\
&= -\partial_x f(x)P_2(x,t) + \partial_x w(x,x(t))g(x(t))g(x)\partial_x P_2(x),
\end{aligned} \tag{A.5}$$

where we have used the identities $f(x(t))\delta(x(t)-x) = f(x)\delta(x(t)-x)$ and $g(x(t))\xi_1(x(t),t)\delta(x-x(t)) = g(x)\xi_1(x,t)\delta(x-x(t))$, and the Furutsu–Novikov formula [31].[3] Noting the identities $w(x,x(t))g^2(x(t))\partial_x P_2 = \partial_x[w(x,x(t))g(x)g(x(t))P_2] - [\partial_x w(x,x(t))]g(x)g(x(t))P_2 - w(x,x(t))[\partial_x g(x)]g(x(t))P_2$, $w(x,x(t))g(x)g(x(t)) = w(x,x)g^2(x)\delta(x-x(t))$, $[\partial_x w(x,x(t))]g(x(t))\delta(x-x(t)) = [\partial_x w(x,x(t))]|_{x(t)=x}g(x)\delta(x-x(t))$, and $w(x,x(t))g(x(t))\delta(x-x(t)) = w(x,x)g(x)\delta(x-x(t))$, equation (A.5) can be written as

$$\partial_t P_2(x,t) = -\partial_x A(x)P_2(x,t) + \partial_x^2 B(x)P_2(x,t), \tag{A.6}$$

in which $A(x) = f(x) + w(x,x)(\partial_x g(x))g(x) + [\partial_x w(x,x(t)]|_{x(t)=x}g^2(x)$, and $B(x) = w(x,x)g^2(x)$.

Since now there is a joint process $(x,\eta(x,t))$ for equation (A.2), the master equations for equation (A.2) should read

$$\begin{aligned}
&\partial_t P(x,t,\xi_1) = -\mu P(x,t,\xi_1) + \mu' P(x,t,a(x)) - \partial_x A(x)P(x,t,\xi_1) + \partial_x^2 B(x)P(x,t,\xi_1),\\
&\partial_t P(x,t,a(x)) = -\mu' P(x,t,a(x)) + \mu P(x,t,\xi_1) - \partial_x[f(x)+g(x)a(x)]P(x,t,a(x)).
\end{aligned} \tag{A.7}$$

Let $P(x,t) = P(x,t,\xi_1)+P(x,t,a(x))$ and $P_1(x,t) = P(x,t,\xi_1) - P(x,t,a(x))$, here $P(x,t)$ is the probability density for equation (A.2). Then, equation (A.7) becomes

$$\begin{aligned}
\partial_t P(x,t) = &-\tfrac{1}{2}\partial_x[A(x)+f(x)+g(x)a(x)]P(x,t) - \tfrac{1}{2}\partial_x[A(x)-f(x)-g(x)a(x)]P_1(x,t)\\
&+\tfrac{1}{2}\partial_x^2 B(x)P(x,t) + \tfrac{1}{2}\partial_x^2 B(x)P_1(x,t),\\
\partial_t P_1(x,t) = &-(\mu-\mu')P - (\mu+\mu')P_1 - \tfrac{1}{2}\partial_x[A(x)-f(x)-g(x)a(x)]P(x,t)\\
&-\tfrac{1}{2}\partial_x[A(x)+f(x)+g(x)a(x)]P_1(x,t) + \tfrac{1}{2}\partial_x^2 B(x)P(x,t)\\
&+\tfrac{1}{2}\partial_x^2 B(x)P_1(x,t).
\end{aligned} \tag{A.8}$$

Now we cannot analytically get the exact expression of the stationary solution of the probability density, even in the natural boundary condition.

Appendix B. A kind of dichotomous noise whose both values are stochastic about *x* and *t*

Here we consider a dichotomous noise $\xi(x,t)$ whose two values are both stochastic. $\xi(x,t)$ takes stochastic variable values $\eta_1(x,t)$ and $\eta_2(x,t)$. We assume that $\eta_1(x,t)$ and $\eta_2(x,t)$ are stochastic Gaussian variables about x and t with zero means and correlation functions $\langle\eta_1(x,t)\eta_1(x',t')\rangle = 2\upsilon_1(x,x')\delta(t-t')$, $\langle\eta_1(x,t)\eta_2(x',t')\rangle = 0$ and $\langle\eta_2(x,t)\eta_2(x',t')\rangle = 2\upsilon_2(x,x')\delta(t-t')$. The transition rates for $\xi(x,t)$ from $\eta_1(x,t)$ to $\eta_2(x,t)$ or vice verse are respectively μ and μ'.

Below we investigate a system driven by this noise. The Langevin equation of this system is

$$\dot{x} = f(x) + g(x)\xi(x,t), \tag{B.1}$$

where $f(x)$ and $g(x)$ are the same as in equation (A.2).

First, we consider the master equations for the noise $\xi(x,t)$. They are

$$\begin{aligned}
&\partial_t P(t,\eta_1(x,t)) = -\mu P(t,\eta_1(x,t)) + \mu' P(t,\eta_2(x,t)),\\
&\partial_t P(t,\eta_2(x,t)) = -\mu' P(t,\eta_2(x,t)) + \mu P(t,\eta_1(x,t)).
\end{aligned} \tag{B.2}$$

[3] The formula obtained by Furutsu and Novikov is $\langle\xi(\mathbf{r})R[\xi]\rangle = \int \mathrm{d}\mathbf{r}'\langle\xi(\mathbf{r})\xi(\mathbf{r}')\rangle\langle\frac{\delta R(\xi)}{\delta\xi(\mathbf{r}')}\rangle$, where $\xi(\mathbf{r})$ is Gaussian with respect to the variable $\mathbf{r}$.

Secondly, let us consider the probability density equations for equation (B.1) when $\xi(x,t)$ takes $\eta_1(x,t)$ and $\eta_2(x,t)$, respectively. When the noise $\xi(x,t)$ only takes $\eta_1(x,t)$, the probability density equation for equation (B.1) is

$$\partial_t P_3(x,t) = -\partial_x A^{(1)}(x) P_3(x,t) + \partial_x^2 B^{(1)}(x) P_3(x,t), \tag{B.3}$$

where $A^{(1)}(x) = f(x) + \upsilon_1(x,x)(\partial_x g(x))g(x) + [\partial_x \upsilon_1(x,x(t))]|_{x(t)=x} g^2(x)$, and $B^{(1)}(x) = \upsilon_1(x,x)g^2(x)$. Equation (B.3) can be obtained as equation (A.6) of equation (A.3). Similarly, we can obtain the probability density equation for equation (B.1) when $\xi(x,t)$ only takes $\eta_2(x,t)$

$$\partial_t P_4(x,t) = -\partial_x A^{(2)}(x) P_4(x,t) + \partial_x^2 B^{(2)}(x) P_4(x,t), \tag{B.4}$$

with $A^{(2)} = f(x) + \upsilon_2(x,x)(\partial_x g(x))g(x) + [\partial_x \upsilon_2(x,x(t))]|_{x(t)=x} g^2(x)$, and $B^{(2)}(x) = \upsilon_2(x,x)g^2(x)$.

So, the master equations for equation (B.1) read

$$\begin{aligned}\partial_t P(x,t,\eta_1) &= -\mu P(x,t,\eta_1) + \mu' P(x,t,\eta_2) - \partial_x A^{(1)}(x) P(x,t,\eta_1) + \partial_x^2 B^{(1)}(x) P(x,t,\eta_1),\\ \partial_t P(x,t,\eta_2) &= -\mu' P(x,t,\eta_2) + \mu P(x,t,\eta_1) - \partial_x A^{(2)}(x) P(x,t,\eta_2) + \partial_x^2 B^{(2)}(x) P(x,t,\eta_2).\end{aligned} \tag{B.5}$$

Let $P(x,t) = P(x,t,\eta_1) + P(x,t,\eta_2)$ and $P_1(x,t) = P(x,t,\eta_1) - P(x,t,\eta_2)$. The probability density $P(x,t)$ for equation (B.1) can be obtained as follows:

$$\begin{aligned}\partial_t P(x,t) = {} & -\tfrac{1}{2}\partial_x(A^{(1)}(x) + A^{(2)}(x))P(x,t) - \tfrac{1}{2}\partial_x(A^{(1)}(x) - A^{(2)}(x))P_1(x,t)\\ & + \tfrac{1}{2}\partial_x^2(B^{(1)}(x) + B^{(2)}(x))P(x,t) + \tfrac{1}{2}\partial_x^2(B^{(1)}(x) - B^{(2)}(x))P_1(x,t),\\ \partial_t P_1(x,t) = {} & -(\mu - \mu')P(x,t) - (\mu + \mu')P_1(x,t) - \tfrac{1}{2}\partial_x(A^{(1)}(x) - A^{(2)}(x))P(x,t)\\ & - \tfrac{1}{2}\partial_x(A^{(1)}(x) + A^{(2)}(x))P_1(x,t) + \tfrac{1}{2}\partial_x^2(B^{(1)}(x) - B^{(2)}(x))P(x,t)\\ & + \tfrac{1}{2}\partial_x^2(B^{(1)}(x) + B^{(2)}(x))P_1(x,t).\end{aligned} \tag{B.6}$$

For equation (B.6), we cannot analytically obtain the exact expression for the stationary solution.

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